LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2007

ST 1810 - ADVANCED DISTRIBUTION THEORY

BB 10

Date : 31/10/2007 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

SECTION – A *Answer **ALL** the questions* *(10 x 2 = 20)*

01. Find the mgf of truncated binomial distribution, truncated at 1.
02. Show that negative binomial distribution is a power series distribution.
03. Find the median of lognormal distribution.
04. Show that the minimum of two independent exponential random variables is exponential.
05. Find the mean of X_1- X_2 when (X_1, X_2) has a bivariate binomial distribution.
06. Let (X_1, X_2) have a bivariate Poisson distribution. Find cov(X_1, X_2).
07. Define bivariate exponential distribution.
08. State the conditional distributions associated with bivariate normal distribution.
09. Let X_1, X_2, X_3, X_4 be independent standard normal variables. Examine whether $X_1-2X_2+3X_3-5X_4$ is independent of $2X_1^2 - 3X_2^2 + X_3^2 + 4X_4^2 - 2X_1X_2 + 10X_1X_3 + 6X_1X_4$.
10. Let X be N(θ,1), θ = 1, 2. If θ is discrete uniform, find the mean of the compound distribution.

SECTION – B *Answer any **FIVE** questions* *(5 x 8 = 40)*

11. State and establish a characterization of geometric distribution based on order statistics.
12. Find the conditional distributions associated with trinomial distribution. Hence show that the regressions are linear.
13. State and establish the additive property of Bivariate Poisson distribution.
14. Derive the mgf of inverse Gaussian distribution.
15. Show that the geometric mean of independent lognormal random variables is lognormal.
16. If (X_1, X_2) is Bivariate exponential, find the cov(X_1, X_2).
17. State and establish any two properties of non-central chi-square distribution.
18. Let X_1, X_2, X_3,…, X_n be independent N(0, 1) variables. If A is an idempotent matrix with rank r, show that $\mathbf{X}' A \mathbf{X}$ is distributed as chi-square with r d.f.

SECTION – C *Answer any **TWO** questions* *(2 x 20 = 40)*

19 a) State and establish the characterization of exponential distribution based on lack of memory property.

b) Given a random sample from inverse Gaussian distribution, show that $\overline{X}$ and $\sum(\frac{1}{X_i} - \frac{1}{\overline{X}})$ are independently distributed.

20 a) Let (X_1, X_2) have a bivariate Poisson distribution. Show that X_1 and X_2 are independent if and only if they are uncorrelated.

b) Show that (X_1, X_2) is bivariate normal if and only if $a_1X_1 + a_2X_2$ is normal $\forall\ a_1, a_2 \in$ R.

21 a) Define non-central t - variable and derive its pdf.

b) Find the mean and variance of non-central F- distribution.

22 a) State and establish a characterisation of bivariate exponential distribution(Marshall – Olkin).

b) Given a random sample from a normal distribution, find the distribution of sample variance using the theory of quadratic forms.

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